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Farey hierarchy in a bimode CO_2 laser with a saturable absorber

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We describe in this paper the dynamics of a bimode CO_2 laser with an intracavity saturable absorber, more precisely the competition between the two modes when their single-mode regime is unstable and as simple as possible, i.e., with a single spike. The transition between the two modes occurs through a succession of periodic regimes separated by nonperiodic ones. The former are governed by the Farey arithmetic and lie on a staircase. Spectrum analyses and return maps suggest the existence of chaos in the nonperiodic states.

The laser is a very interesting illustration of the richness of the dynamical phenomenology of nonlinear systems, in particular concerning the phenomena connected with chaos.[1] The CO_2 laser has received special attention, and has been investigated extensively, in different configurations.[2-8] Evidence of homoclinic chaos was demonstrated in the CO_2 laser with feedback,[5] as well as in the CO_2 laser containing a saturable absorber (LSA).[6] The latter is particularly interesting since it is an all-optical system where the feedback is intrinsic.

We report in this paper on the experimental investigation of a bimode CO_2 LSA. We concentrate here on some basic results obtained with bimode operation in the simple case where both modes exhibit periodic pulses without structure. We show that in this case, the dynamics may be interpreted as the competition of two instability modes that interact following the Farey arithmetic, in a sequence of periodic regimes separated by stochastic ones that present properties of deterministic chaos.

The experimental setup is made of a CO_2 laser containing CH_3I as a saturable absorber. The essential modification of the setup previously described in Ref. 6 consists of an increase of the transverse size of the laser cavity to allow for two-mode operation. The laser oscillates on the $P(32)$ line of the 10.6-μm branch. The modes involved here are two transverse modes separated by 15 MHz. This frequency has to be compared with the free spectral range (50 MHz) and the cavity linewidth (1 MHz). The output intensity of the laser is detected through a $Hg_xCd_{1-x}Te$ detector that is sensitive to the total intensity of the laser. We used here as a control parameter the length of the cavity that governs the frequency detunings between the laser transition and both cavity modes.

The behavior of the single-mode CO_2+CH_3I LSA was extensively studied in Ref. 6. In our experimental conditions, it is characterized by a self-pulsing regime called passive Q switching (PQS) in which the time evolution of the laser intensity exhibits very different shapes depending on the operating point.[9] We limit this paper to the case of the competition of two modes whose single-mode operation is the simplest unstable one, i.e., $P^{(0)}$ following the terminology introduced in Ref. 9. This means that the laser emits periodic single-spike pulses. The $P^{(0)}$ regimes of the two modes differ by both the period and the peak power of their pulses, that depend on the mode characteristics, such as the central frequency or the losses. We have checked by a heterodyne detection that in our experimental conditions, the peak power of each mode in the unstable regime remains approximately constant as the cavity length is changed. For the sake of clarity, we name mode 1 (mode 2) the mode with larger (smaller) peak power.

As the cavity length is increased, the laser oscillates first on mode 1 only, then in a bimode way, and eventually on mode 2 only. In the range where the two modes interact, the signal consists of a succession of antiphase oscillations of the two modes, as described in Ref. 10. The transition from pure mode 1 to pure mode 2 as the cavity length is changed appears as a succession of periodic states separated by nonperiodic regimes.

Let us first examine the sequence of periodic regimes. In the following, we will call r^p a periodic regime formed by p large pulses similar to those emitted by mode 1, followed by r small pulses similar to those of mode 2. With this notation, the single mode 1 operation is noted 0^1 and the corresponding one for mode 2 is 1^0. A similar notation was used by Maselko and Swinney in their study of the Belousov-Zhabotinski reaction.[10] Figure 1 shows as an example a sequence of periodic oscillations observed as the cavity length is increased. Two kinds of regimes may be distinguished: (i) the basic ones simply coded by r^p, as the 1^1 or the 2^3 regimes, and (ii) those made by concatenation of two or more basic regimes $r_1^{p_1}$ and $r_2^{p_2}$, in the form $(r_1^{p_1})^n(r_2^{p_2})^m$, as, e.g., $(1^2)^2 2^2$.

A "firing number" has been associated with each periodic state, following the procedure proposed in Ref. 10. The firing number of an r^p regime is defined as p/q with $q=r+p$. For example, the firing numbers associated with 1^1, 2^2, and 2^3 are $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{3}{5}$, respectively. It appears here that different regimes may have the same firing number. The firing number of a multipatterned signal is obtained by using the Farey arithmetic, where the sum $\oplus$ of two rational numbers p_1/q_1 and p_2/q_2 is

$$\frac{p_1}{q_1} \oplus \frac{p_2}{q_2} = \frac{p_1+p_2}{q_1+q_2} .$$

Figure 2 shows the succession of periodic states ob-

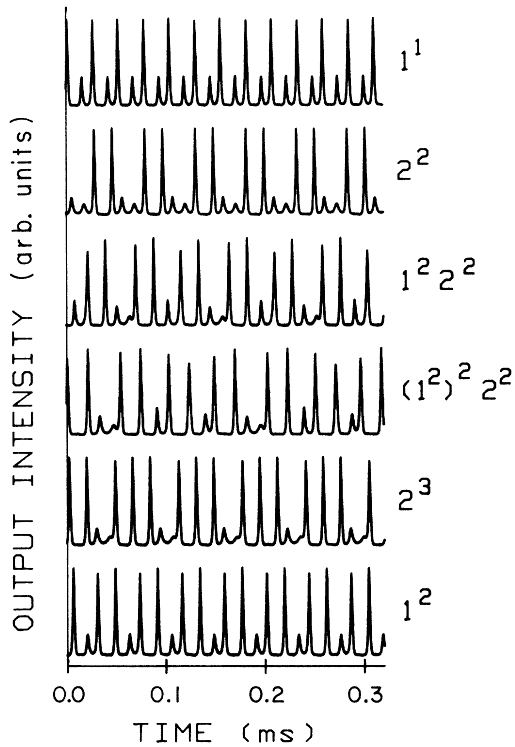

FIG. 1. Examples of periodic regimes observed between the 1^1 and the 1^2 states.

served in our experiment when the frequency detuning is varied so that the laser changes from mode 2 to mode 1 from left to right in the figure. On the y axis the firing number associated with each observed periodic regime is reported. It appears that they are organized following a staircase, where the changes in period occur through a monotonic variation of the firing number.

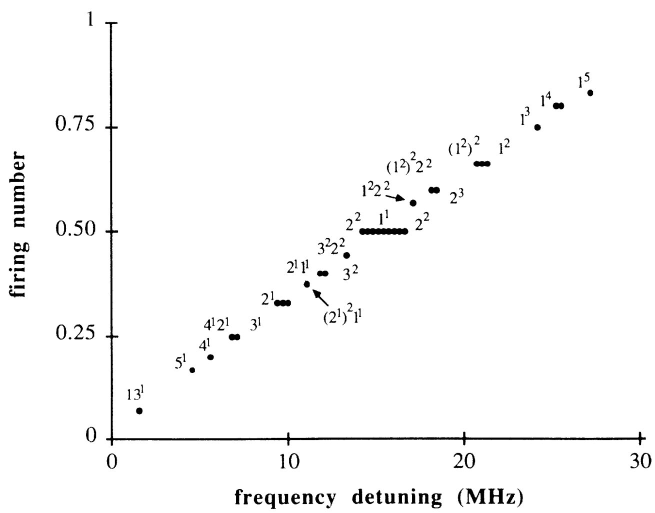

FIG. 2. Plot of the firing number of all observed periodic states as a function of the cavity detuning. The origin of the detuning was fixed arbitrarily at the appearance of mode 1 (border between pure mode 2 and bimode regimes, i.e., right side of the step 1^0).

It must be noticed that the different basic periodic states observed in our experiment may be deduced using the Farey arithmetic from the two limit regimes 1^0 and 0^1. The Farey addition of the firing numbers associated with the latter gives $\frac{0}{1} \oplus \frac{1}{1} = \frac{1}{2}$, the simplest state associated with it is 1^1. The next steps of the tree may be determined in the same way. All the basic regimes of the first four levels of the tree have been observed in our experiment, as reported in Fig. 2, together with some states of higher levels. Other ones would be undoubtedly observed by increasing the stability of the experiment. As it was noticed above, each firing number may be associated with several regimes. For instance, 2^2 takes in the tree the same place as 1^1. The multipatterned regimes follow the same arithmetic. Thus, $1^2 2^2$ is equivalent in the tree to 3^4, corresponding to a firing number of $\frac{4}{7}$. Every observed regime may be placed in the tree, and follows the Farey hierarchy.

The plot of Fig. 2 has a staircase structure that is strikingly similar to that encountered in the evolution towards chaos of quasiperiodic systems.[11] Here, the number of steps is finite and the staircase is not exactly the devil's

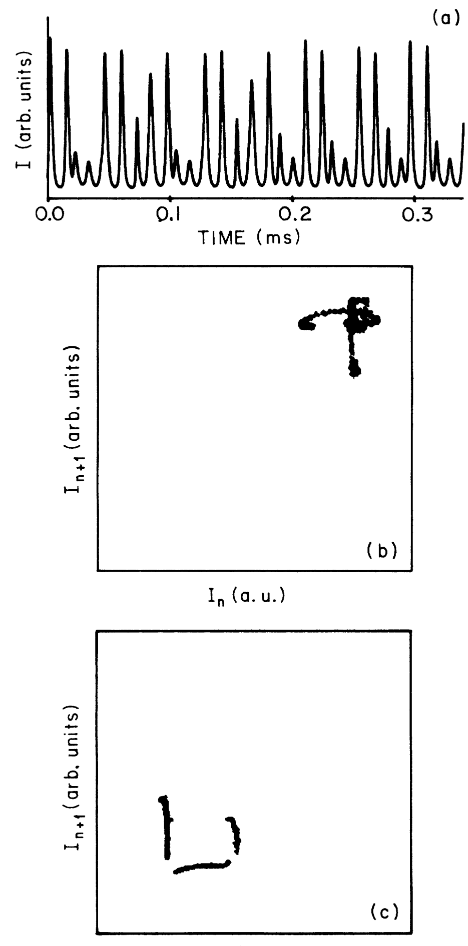

FIG. 3. Chaotic regime observed between the 1^1 and 2^2 regimes. In (a), total intensity of the laser as a function of time. In (b) [(c)], first return map (I_n, I_{n-1}) for intensity of mode 1 (mode 2).

staircase which is obtained, e.g., at criticality in the circle maps.[11]

Nonperiodic regimes appearing between the periodic regions do not correspond to quasiperiodicity. In these regimes, the spectrum of the laser intensity revealed the appearance of a new frequency in the system, accompanied by a growth of the continuous background. Although the second frequency seems to imply the presence of a torus, the general shape of the spectrum was rather that of a chaotic signal. A more accurate analysis was performed through the reconstruction of the attractor in a derivative phase space $(I,\dot{I},\ddot{I},\dots)$. In such a phase space, a plot of the nth maxima versus the $(n-1)$th ones is a first return map. As the total intensity is not a simple variable of the system, the above treatment was applied separately to the intensity of the two laser modes. Figure 3 shows the return maps of mode intensities in the nonperiodic regime occurring between 1^1 and 2^2. A close look to them indicates that the part of the cycle that appears, e.g., in the upper right corner of Fig. 3(b) is not a cycle truncated by a limited number of points, but keeps its shape as the number of points is increased with an accumulation point at one end. This suggests, together with the existence of a broad spectrum, that this nonperiodic regime is chaotic. Similar results were obtained in different points of the staircase.

We have shown that the LSA, in which two modes in their single-spike pulsed regimes interact, exhibits antiphase oscillations. The transition between the two modes occurs through a succession of periodic states separated by chaos. We have shown that the whole transition between the two modes as a function of the laser cavity length, is governed by the Farey arithmetics if we associate with each state a firing number deduced from a simple code. Future experimental investigations should refine the study of nonperiodic regimes by using higher dimensions to reconstruct the attractor and defining a local measure to plot one-dimensional maps. A good understanding of the mechanisms from which this behavior originates should be obtained by changing the parameters of the system. However, a simple change of the well-controlled parameters leads here to the appearance of monomode regimes with more complicated line shapes, e.g., $P^{(n)}$-type oscillations.[9] As a consequence, a fruitful exploration of the parameter space appears to be difficult in such experiments.

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