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Homoclinic orbits and cycles in the instabilities of a laser with a saturable absorber

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The phase-space evolution for the instabilities in a CO_2 laser with an intracavity saturable absorber is investigated experimentally. The different scenarios corresponding to limit cycles, homoclinic orbits and cycles involving two unstable points, and chaotic behavior are investigated. A theoretical analysis of the experimental results is sketched out.

A laser containing an intracavity saturable absorber (LSA) is a simple but prolific nonlinear optical system. By an appropriate choice of the control parameters, instabilities occur in the LSA operation and the laser output power appears composed by spikes or modulations periodic in time and, as shown in this paper, eventually leading to chaotic behavior. The instabilities are associated respectively to the installation of hard modes or of subcritical Hopf bifurcation. The spiking regime is known as the passive Q switch (PQS) from the early days of the CO_2 LSA operation.[1] The modulated regime has been reported in more recent LSA observations.[2] Under an appropriate choice of the laser parameters, not corresponding to experimental ones, the appearance of both kinds of instabilities has been explored in LSA theoretical analyses based on the Maxwell-Bloch and rate-equation approaches.[2–4] However, for the range of parameters accessible in LSA experiment, no theoretical analysis has defined precisely the occurrence of instabilities and chaos in an experimental configuration.

An analysis of LSA instabilities based on the phase-space description is here reported: both experimental observations and theoretical investigations are presented. A detailed analysis of the instabilities defined as type-I, PQS-like, and type-II, high-frequency modulation, is presented. LSA equations have two fixed points: an I_0 solution corresponding to a zero laser output power and an I_+ solution corresponding to a laser output power different from zero. In the instability regime here considered, both fixed points are unstable, with a saddle point in I_0 and a saddle focus in the I_+ point. The main result of this paper is that the different LSA instabilities are represented by orbits in the phase space influenced by either one unstable point or both unstable points. The type-I instability is associated to either an homoclinic orbit connecting the I_0 point and revolving around the I_+ point, or an homoclinic cycle joining the two unstable fixed points (we follow here definitions of Ref. 5). More precisely the phase-space evolution should be described on the basis of quasihomoclinic orbits or cycles. Type-II instabilities correspond to limit cycles around the I_+ unstable point with smaller or wider orbits in the phase space. The main feature of homoclinic orbits and cycles, in contrast to the wide orbits in type-II instabilities, is that the phase-space homoclinic trajectories approach the I_0 point at a slow speed and remain in its neighboring for a long time as compared to the time spent on the trajectory. An important result of the present paper is the observation that chaos in the LSA may be reached through a cascade of period-doubling bifurcations on the type-II instability.

This work was prompted by a recent investigation of the occurrence of homoclinic orbits and Shil'nikov chaos in an internally modulated laser with overall feedback as illustrated by a phase-space analysis.[6] The homoclinic orbits and cycles representing the evolution of the I laser amplitude in the LSA are related to the motion around a saddle focus. A phase portrait of the motion in the $(I,\dot{I})$ space described by a homoclinic cycle for the laser amplitude is schematically represented in Fig. 1. By a wide orbit, the system moves away from the neighboring of the I_0 saddle point converging towards the neighboring of the saddle focus I_+ and emerging out from this point with an outwards spiraling motion. A similar orbit has been called an homoclinic reinjection process for a chemical reaction[7] or an inverse Shil'nikov process for the LSA.[8] In the usual Shil'nikov chaotic configuration, the number of outwards spirals of the trajectory presents a sensitive dependence upon the initial coordinate of the approach to the saddle focus. On the contrary, in the LSA the simultaneous presence of a strong attractive saddle point stabilizes the orbits and does not lead to a Shil'nikov chaos. Our experimental and theoretical observations on an all-optical autonomous system, such as the LSA that contains two unstable points with a particular relation between the attractive and repulsive parts of the eigenvalues, are of particular relevance in the aim of characterizing instabilities and

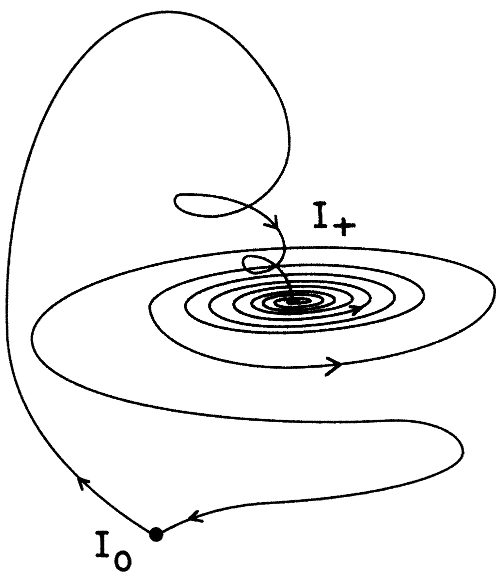

FIG. 1. Schematic representation of a LSA homoclinic cycle with two unstable points I_0 and I_+.

chaos in nonlinear optical systems.

In the experimental setup the output power of a single-line, single-mode, homogeneously broadened infrared CO_2 laser containing an intracavity low-pressure (10–300 mTorr), SF_6, CH_3I, or CF_3Br absorber gas, and buffer He gas is monitored as a function of the CO_2 discharge current or the laser cavity length.[9] The discharge current modifies the A pump control parameter, i.e., the ratio of the amplifier unsaturated small gain to the cavity losses. The laser cavity length modifies the two frequency detunings, i.e., the differences between the laser operation frequency and the amplifier and absorber frequencies. The absorber pressure affects the absorber control parameter and the relative saturability of the amplifier and absorber.

The time-dependent laser output power was recorded by a transient digitizer and transferred to a microcomputer for the construction of the laser intensity return map and of the statistical distribution and the return map of return times. Phase portrait for the laser amplitude in the plane $(I,\dot{I})$ was applied to monitor the laser evolution. The phase portrait topologic description monitors the global character of the laser amplitude in the instabilities and its correspondence to the fixed points and the space structure. The return-time observations permit the singling out of the characteristic behavior of the period bifurcation-type or Shil'nikov-type chaos or of the noise presence.[6(b)] In fact the large spread in the return time is a distinctive feature of the homoclinic chaotic behavior with fluctuations in the revolution times.

In the LSA instabilities the zero laser intensity I_0 solution and the nonzero laser intensity I_+ solution are unstable in an interval of control parameters that at fixed laser detunings and absorber pressure is denoted by the A pump parameter. The instability interval is limited by two bifurcation extreme points, one of them being the A_H Hopf bifurcation point. Different homoclinic bifurcations may appear within the instability interval. Depending on the locations of these homoclinic bifurcations in comparison with the A_H point, different scenarios are observed experimentally, with coexistences between them when the control parameters are continuously tuned.

A first scenario contains type-I instabilities with a time-dependent laser output power equivalent to a quasihomoclinic orbit of the point I_0 visiting a large region of the phase space, as in the case of Fig. 2(a). Shifting the laser operation point, the homoclinic orbit breaks down progressively, and the I_+ attraction becomes stronger. Thus the system, before coming back in the I_0 neighboring, may fall on the unstable manifold of the I_+ point, leaving the I_+ neighboring through revolutions on an almost plane-divergent spiral. Depending on the phase-space distance between the I_0 unstable manifold and the reinjection point into the I_+ unstable plane, the revolutions around I_+ remain wide, or may be very small, leading finally to a heteroclinic connection between I_0 and I_+, i.e., to a total quasihomoclinic cycle, as in Fig. 2(b).

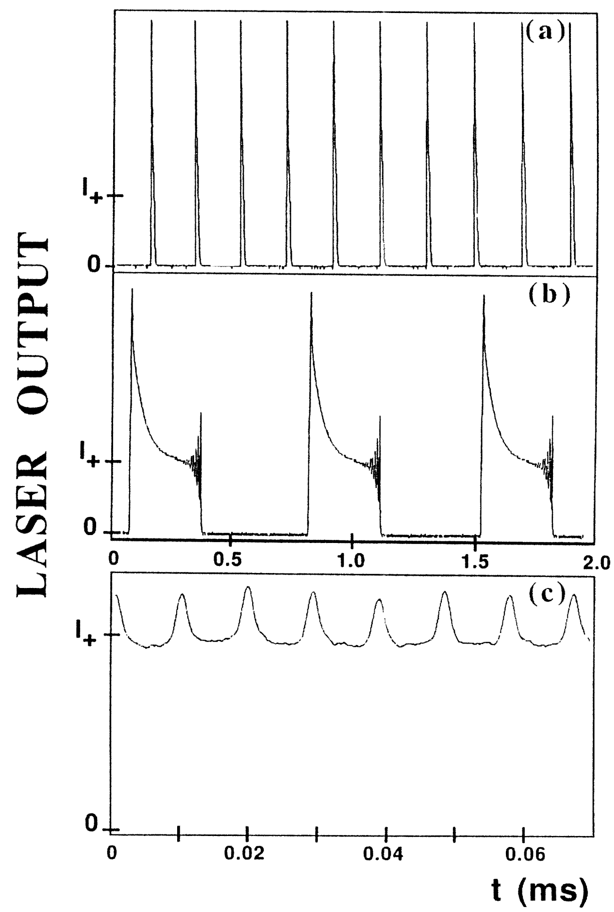

FIG. 2. Pulses of output power (in arbitrary units) vs time (in ms) for different laser instabilities. In (a) and (b) type-I instabilities pulses corresponding, respectively, to an homoclinic orbit of I_0 and to an homoclinic cycle connecting I_0 and I_+, with LSA operation on the $10P(24)$ CO_2 laser line containing 25 mTorr SF_6 gas pressure at different pump powers. In (c), a type-II instability corresponding to an orbit around I_+; LSA operation in the bistable regime on the $10P(16)$ CO_2 line with 16-mTorr SF_6 absorber.

When the laser operation moves towards the A_H bifurcation point, the repulsive eigenvalues become smaller and the number of spirals increases. The phase portrait of Fig. 3, corresponding to the experimental conditions of Fig. 2(b), illustrates very clearly the time evolution for the homoclinic cycle. Notice that the addition of He as buffer gas in the experimental records of Fig. 2 allows the monitoring of, over the frequency tuning of the laser mode, a complete scenario concerning the number of revolutions made by the orbit in screwing to and spiraling out of the I_+ saddle focus. The transition from an orbit with n spirals to an orbit with $n+1$ spirals takes place discontinuously on the control parameter; i.e., only complete circular revolutions around I_+ are observed on the time evolution. However, for an LSA operation close to a transition in the number of spirals, our experimental observations have shown the presence of hesitations of the system between two alternative orbits differing in the number of spirals. These hesitations are originated by the internal noise as quasihomoclinic trajectory are very sensitive to the system noise.[6(b)] This noise dependence was monitored in our experiment by observing the large modifications produced by an externally added noise on the trajectory around I_+ and on the return time map.

In the second scenario the type-II instabilities, characterized by phase-space orbits centered around I_+, are illustrated by the record of Fig. 2(c) for an orbit with a medium amplitude around the I_+ unstable point. Type-II instabilities have a period typically very close to the repetition period of the spirals observed in the homoclinic cycles of type-I instabilities. The laser pulses of Fig. 2(c) present a large fluctuation in the amplitude corresponding to a low-frequency noise in the frequency spectrum, again evidence of the large influence by the noise on the LSA evolution around the I_+ unstable point. In the experimental observations the transition from the first to second scenario takes place discontinuously on the laser detuning with an asymmetric behavior in respect to the center of the cavity mode.

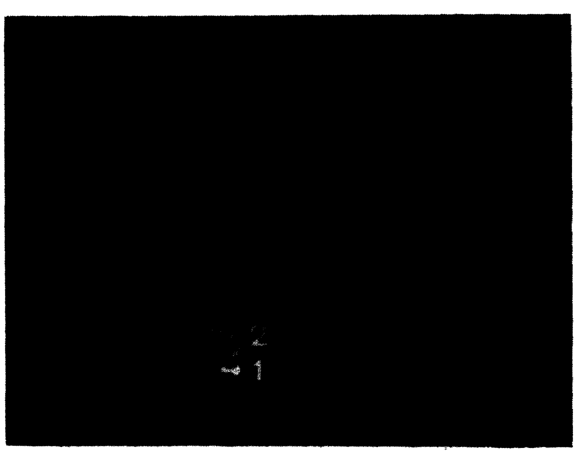

FIG. 3. $(I,\dot{I})$ phase portrait for the homoclinic cycle reported in Fig. 2(b). On the photo, 1 and 2 correspond to the I_0 and I_+ unstable points. In the phase portrait the system spends a lethargic time in the neighboring of the I_0 point, and leaves the I_+ point through a series of spirals.

In our study of the LSA instabilities we have carefully looked for evidences of chaotic behavior. By a proper setting of the laser frequency tuning it was possible to obtain a period-doubling bifurcation diagram on the type-II instabilities. Periods up to 8 T have been observed and an example of a chaotic behavior is reported in Fig. 4(a). No example of Shil'nikov chaos was obtained on the type-I homoclinic orbits or cycles and the observed spread in the return time was associated to the noise influence on the system evolution around the fixed points. However, a period-doubling bifurcation, with pulses very regular in time, but alternating on the amplitude, could be observed on the type-I instabilities, as reported in Fig. 4(b). Notice that Figs. 4(a) and 4(b) were obtained for the same LSA conditions except for the cavity frequency detuning, a very convenient control parameter to explore the instability bifurcation points.

The experimental observations evidence the important role played by the detuning parameters in the instability scenario. In the theoretical analysis of Ref. 3 with detunings included in the Maxwell-Bloch LSA equations, laser pulses with spiraling motion out of the I_0 solution were obtained but a screw or spiral motion around the I_+ point was not. On the contrary, our numerical analysis based on the rate-equation model introduced by Tachikawa, Tanii, Kajita, and Shimizu[10] to describe the pumping mechanism in the CO_2 laser from the ground vibrational state to the upper lasing vibrational state was able to

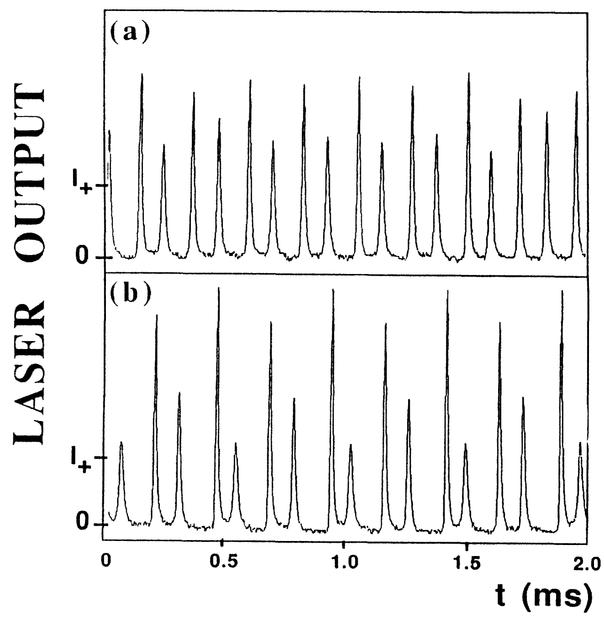

FIG. 4. (a) Chaotic behavior on the LSA output pulses reached through a period-doubling bifurcation sequence for the type-II instability. (b) Period doubling observed on type-I instability, homoclinic orbit from I_0. LSA operation on the $10P(32)$ CO_2 laser line with 100 mTorr SF_6 absorber and 900 mTorr He buffer gas at $A = 1.2$ was used in both records with zero cavity detuning in (a) and 5 MHz cavity detuning in (b).

reproduce the observed behavior associated to the homoclinic orbits and cycles. The coexistence of different scenarios was not reproduced in our model. We have numerically explored the instabilities also in an improved model including the rotational structure of the amplifier and absorber and presenting a much better agreement with the experimental results. In the Tachikawa *et al.* model, the system is described in a four-dimensional space. The fixed I_0 point, when unstable, is a saddle one, that means four real eigenvalues: one positive and three negative. I_+ is a saddle focus with two real negative eigenvalues λ_i $(i=1,2)$ and two complex-conjugate eigenvalues $\rho \pm i\omega$. The LSA behavior depends on the relative value of these eigenvalues. The condition $|\rho/\lambda| < 1$, a necessary and sufficient condition for the occurrence of Shil'nikov chaos in the case of a single unstable point presenting an homoclinic orbit,[5] is verified for our experimental conditions. However, we have noticed that for the parameters corresponding to the experiment, the repulsive ρ eigenvalues are systematically smaller than the attractive eigenvalues at the I_0 saddle point, making the I_0 point strongly attractive and preventing the orbit around it to grow enough for the occurrence of a chaotic regime of the Shil'nikov type.

LSA instabilities characterized by the presence of two unstable points are a special case of autonomous nonlinear optical systems to be investigated in more detail as concerns the relation between the global behavior and the local eigenvalues. Hesitations are a new feature presented by this system, and the relation between them and the occurrence of chaos in a small region of control parameters should be explored more.

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[1]P. L. Hanst, J. A. Morreal, and W. Henson, Appl. Phys. Lett. **12**, 58 (1968); A. G. Fox, S. E. Schwartz, and P. Smith, *ibid* **12**, 371 (1968).

[2]E. Arimondo, P. Bootz, P. Glorieux, and E. Menchi, J. Opt. Soc. Am. B **2**, 93 (1985).

[3]J. C. Antorantz, L. L. Bonilla, J. Sea, and M. G. Velarde, Phys. Rev. Lett. **49**, 35 (1980).

[4]T. Erneux, P. Mandel, and J. F. Magnan, Phys. Rev. A **29**, 2690 (1986).

[5]J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields* (Springer, Berlin, 1984).

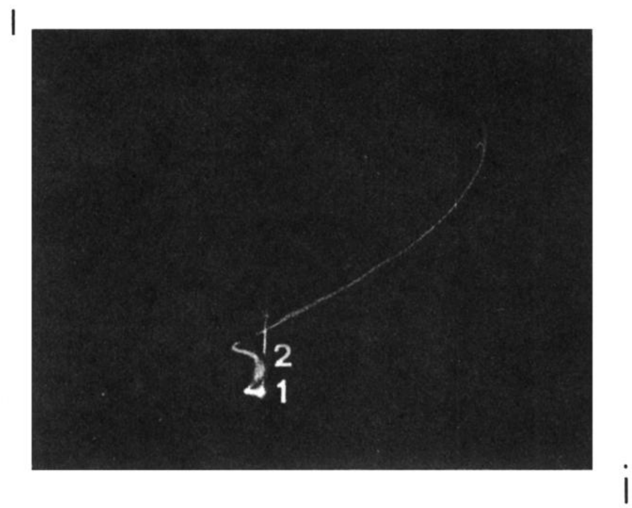
[6](a) F. T. Arecchi, R. Meucci, and K. Gadomski, Phys. Rev. Lett. **58**, 2205 (1987); (b) F. T. Arecchi, R. Meucci, J A. Roversi, and P. H. Coullet, Phys. Lett. (to be published).

[7]F. Argoul, A. Arneodo, and P. Richetti, Phys. Lett. A **120**, 269 (1987).

[8]D. Dangoisse, A. Bekkali, F. Papoff, and P. Glorieux, in *Technical Digest International Workshop on Instabilities, Dynamics and Chaos in Nonlinear Optical Systems,* edited by N. B. Abraham, E. Arimondo, and R. W. Boyd (ETS Editrice, Pisa, 1987); and (unpublished).

[9]E. Arimondo, D. Dangoisse, C. Gabbanini, E. Menchi, and F. Papoff, J. Opt. Soc. Am. B **4**, 892 (1987).

[10]M. Tachikawa, M. Tanii, M. Kajita, and T. Shimizu, Appl. Phys. B **39**, 83 (1986).



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